Introduction
We solve numerically the system of Ginzburg-Landau equations describing the superconductor containing Abrikosov vortices in the special geometry essential for practical application: bulk material with a cylindrical hole.

Ginzburg-Landau equations
\[
\left( \frac{j}{\kappa} \nabla + A \right)^2 \Phi - \Psi^* \Phi + |\Psi|^2 \Phi = 0, \\
\nabla \cdot (\Phi \nabla \Psi - \Psi \nabla \Phi) + |\Psi|^2 A = 0,
\]
where \(\Psi\) and \(A\) are the wave function of Cooper pairs and vector potential of the magnetic field. Boundary conditions on the hole radius
\[
\left( \frac{j}{\kappa} \nabla + A \right) e_\theta \Psi = 0, \\
(\nabla \times A)_r |_{r=R} = H_z,
\]
and Coulomb gauge for vector potential:
\[
\nabla \times A = 0.
\]

Numerical solution
- Reformulation of equations for vortex problem[1]:
  \[
  \Phi(r, \theta) = \Phi_i(r) \cdot \Phi_i(r - \rho_c), \\
  A(r, \theta) = A_i(r) \cdot A_i(r - \rho_c),
  \]
  where the 2D coordinate \(\rho_c = (x_c, y_c)\) denotes the center of the vortex.
- Single vortex solution (example for Pb, \(\kappa = 0.45\))

OpenCL application
1. Iterative approach allows us to use the Production-CL library for iterative scientific calculations [2].
   - Batch of numerical kernels: inside_2D(), boundary_conditions_1D(), time_step_2D()
   - Use max. number of iterations, numSave between saving the state, numSelf between recalculation of currents and magnetic field (self-consistent solution)
   - possibility to continue the calculation from the saved state
2. During calculation (iterations):
   - simulation of main equations and time step: \(N_x \times N_y\) points, 3 components \((A_r, A_\theta, \Phi)\) in parallel;
   - applying boundary conditions: \(N_x\) points, 3 components \((A_r, A_\theta, \Phi)\) in parallel using FFT;
   - parallel calculation of electrical current inside the superconductor volume: \(N_x \times N_y\) points, 2 components \((j_r, j_\theta)\) in parallel;
   - self-consistent magnetic field on hole surface: \(N_y\) points, 1 component \(H_z\), each needs integration of \(N_x \times N_y\) terms (parallel reduction);
3. After convergence, to obtain the value of Gibbs free energy:
   - calculate magnetic field inside hole: \(N_x \times N_y\) points, \(N_x\) terms integration of \(N_x \times N_y\) terms;
   - calculate the value of Gibbs free energy: integrate 3 terms – magnetic field energy inside the hole, \(|\psi|^4\) and \(|\nabla \psi|^2\) inside the superconductor. \(N_x \times N_y\) terms, parallel reduction.

OpenCL/GPU specific tricks
- local memory storage of data when calculating finite-difference derivatives
- immediate in-place calculation of nonlinear coordinate metrics, instead of reading from memory
- several simulations for different parameters simultaneously
  to maximize GPU occupancy

Results of simulation
1: Vortex pinning, 2: Surface curvature, 3,4: convergence control
As expected, we see the potential well profile \(U(d)\) for the so-called vortex pinning on the hole, depending on the hole radius \(R\).

Speed-up
![Graph showing speed-up results](image)

Aknowledgements
The author is grateful to Prof. V.A.Kashurnikov for the initial problem formulation, and to Prof. A.I.Podlivaev for valuable hints regarding the numerical approach.
The work in supported by the Russian Foundation for Basic Research, Grant No. 18-02-00278.

Bibliography