OPENCL™ FAST FOURIER TRANSFORM OPTIMIZATIONS FOR INTEL® PROCESSOR GRAPHICS

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Presenter: Michal Mrozek

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Agenda

• Introduction
• Implementation overview
• Optimizations
• Results
• Conclusion and future work
What is Fast Fourier Transform (FFT)?

**DTFT**

\[ X(f) = \sum_{-\infty}^{+\infty} x(t) e^{-2\pi j ft} \]

**DFT**

\[ X_k = \sum_{n=0}^{N} x_n e^{-\frac{j 2\pi n k}{N}} \]

**FFT**

\[ X_k = \sum_{n=0}^{N/2} x_{2n} e^{-\frac{j 2\pi 2n k}{N/2}} + \sum_{n=0}^{N/2} x_{2n+1} e^{-\frac{j 2\pi (2n+1) k}{N/2}} \]

- Discrete-Time Fourier Transform (DTFT)
  - Infinite series, can’t be computed
- Discrete Fourier Transform (DFT)
  - A sampling of the DTFT
- FFT
  - A fast way to compute DFT

Cooley-Tukey radix 2 decomposition

1. \( t = n \times T \), \( T \) = quanta of time
What is Fast Fourier Transform (FFT)?

- Discrete-Time Fourier Transform (DTFT)
  - Infinite series, can’t be computed
- Discrete Fourier Transform (DFT)
  - A sampling of the DTFT
- Fast Fourier Transform
  - A fast way to compute DFT

1 Here the signal is actually complex, the chart shows magnitude.
2 Not really a DTFT, just an approximation... DTFT can't be computed.
Typical Ways to Compute Fast Fourier Transform (FFT)

Cooley-Tukey
- Recursively split the FFT into smaller equal sizes

Split-Radix
- Radix 4:2:2

Stockham
- Eliminates the need for rearranging the inputs/outputs that is specific to Cooley-Tukey

Prime-factor
- Decompose into relatively prime numbers

Bluestein (Chirp-Z)
- For arbitrary sizes, uses Cooley-Tukey and convolution theorem

many others...
Main Differences versus Other Implementations

- No shared local memory, no barriers
  - Maximize device occupancy
  - Reduced code complexity
- Column major in/out
  - Intended for progression to 2D Fast Fourier Transform (FFT)
  - Maximizes memory bandwidth (GB/s)
  - Cache-friendly
- FFT decomposed into smaller FFTs
  - Called here “base FFTs”
  - Maximize register use
  - Reduced code complexity
- Multi-kernel
  - Each base FFT as a separate kernel
- Code generation for
  - Any local/global size configuration
  - Any register size/SIMD size
genFFT Code Generator and Execution Flow

- Decompose the Fast Fourier Transform (FFT) into factors power of two
  - Base FFTs must maximize register use
  - Minimize the difference of the last two factors – best performance
- Generate each base FFT
  - Local twiddle factor lookup table (LUT) in registers
- Generate intermediary twiddle factors
  - LUT in global memory
- Generate bit rotations
  - In order to unscramble the data at the end

\[ N = \prod_{q=0}^{Q} N_q \]

- Read from Global Memory
- \( N_q \) Base FFT
- \( q=Q-1 \)
- Intermediary Twiddle Factors
- Write to Global Memory
- Out-of-place Bit Rotations
- Done
Base Fast Fourier Transform (FFT) Butterflies

- Cooley-Tukey radix 2 butterflies
  - 8-Point DIT FFT on the right
    - Decimation-In-Time

- Pseudo-code
  1. Read signal
  2. Bit reversal
  3. Perform the butterflies
     - Apply twiddle factors
  4. Write spectrum

\[ X_k = \sum_{n=0}^{N/2} x_{2n} e^{-j\frac{2\pi}{N/2} 2nk} + \sum_{n=0}^{N/2} x_{2n+1} e^{-j\frac{2\pi}{N/2} (2n+1)k} \]
Base Fast Fourier Transform (FFT) Bit Reversal

- Each base FFT performs its own bit reversal
  - No global bit reversal at the beginning
  - A distributed bit reversal strategy

- Advantages
  - The preprocessor eliminates bit reversal math at compile time
  - The input is read directly into the correct registers

- Disadvantages
  - This still requires bit rotations at the end of a multi-kernel FFT pipeline

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1 It can be argued that the bit rotations exhibit a more regular memory access pattern than a global bit reversal.
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Multi-Kernel Fast Fourier Transform (FFT) Implementation

- Cooley-Tukey decomposition into factors power of 2
  - Each factor treated as a radix-2 Cooley-Tukey FFT with a different stride
  - Each FFT as an independent kernel—base FFT
  - Base FFTs make efficient use of registers
  - Each base FFT performs its own bit reversal on inputs
    - Intermediary twiddle factors
    - Bit rotations required at the end

\[
FFT_N = FFT_{N_1} \left( W_{N_1}^{n_1 k_2} FFT_{N_2} \right)
\]

\[
N = 64 = 8 \times 8
\]
Multi-Kernel Fast Fourier Transform (FFT) implementation

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Optimization Notice
Base FFT Twiddle Factors Lookup Table

- Storage requirements reduced from N/2 cl_float2 values to N/4+1 cl_float values
- Private array lookup table small enough to fit into registers

\[ W_N^k = e^{-j\frac{2\pi k}{N}} = \cos(-2\pi k/N) + j \sin(-2\pi k/N) \]
Base FFT Twiddle Factors Lookup Table

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\]
Base FFT Twiddle Factors Lookup Table

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<th>sin(i)</th>
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constant float W[9] =

{ +0.0f,
  +0.19509f,
  +0.38268f,
  +0.55557f,
  +0.70711f,
  +0.83147f,
  +0.92388f,
  +0.98078f,
  +1.0f,
};

cos(k) = +W(FFT_SIZE/4-FFT_SIZE/crtFFT * k);

sin(k) = -W(FFT_SIZE/crtFFT * k);

cos(k) = -W(FFT_SIZE/crtFFT * k);

sin(k) = -W(FFT_SIZE/4-FFT_SIZE/crtFFT * k);
Intermediary Twiddle Factors Lookup Table

- Intermediary twiddle factors: \( W_N^{n_1n_2} = e^{-j\frac{2\pi}{N}n_1n_2} \)
  - Different than the base FFT twiddle factors
  - Total of N cl_float2, quite large
  - We reduced it to N/4+1 cl_float values (8x)
    - At the cost of some pretty complicated indexing math
  - Can be further reduced to half of that
    - At the cost of extra math (sqrt) and accuracy
  - >10 percent performance loss
  - In the end: global array of N cl_float2 kernel argument

\[ N = 64 = 8 \times 8 \]

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<td>62</td>
<td>63</td>
<td></td>
</tr>
</tbody>
</table>

FFT 8 stride 8
FFT 8 stride 1

* Intermediary Twiddle Factors
Efficient Use of the Execution Unit Registers

- 7 hardware threads
- 128 registers per hardware thread
  - 32 bytes per register
- 512 bytes per work item
  - In SIMD 8 mode
- \( N_r = 32 \), max FFT size that fits in registers
  - Up to 64 cl_float2 per work item
  - leave room for other program variables

[Junkins 2014-2015]
Efficient Use of Cache and Memory Bandwidth

- **Goals**
  - Reduce number of cache lines touched
  - Reduce the number of memory address requests that an execution unit makes to the data port
  - 90 percent of peak last level cache bandwidth (GB/s)
  - For base FFT kernels
  - On Intel® Processor Graphics we recommend reading up to 4x32-bit data per work item
    - 2x32-bit was a good balance of performance and code complexity
    - Input signal in column major order
Improving Kernel Performance

• Eliminate the use of shared local memory and barriers
  • Maximize device thread occupancy
  • Reduced code complexity
  • Code can be tuned for various architectures

• Perform bit reversal in registers
  • Reduced penalty
  • Preprocessor eliminates index math

• Math transcendental vs lookup table (LUT)
  • For the intermediary twiddle factors
  • Math transcendental require FRM for performance → poor accuracy
  • LUT in global memory
    • FRM-agnostic
    • <10 percent slower than sin/cos
    • Very good accuracy
Improving Kernel Performance

• Fuse two or more kernels
  • Straightforward for two consecutive base FFTs of the same length:
    • 8x8, 16x16, 32x32
  • Kernels become function calls with a barrier in between
  • Workgroup size increased to account for data dependencies
  • Can do 64-point FFT and 256-point FFT but can’t generalize to any FFT length

• Increase work per work item
  • Straightforward change due to the reduced complexity of our code
  • By itself it doesn’t lead to performance gains
  • 2x work per work item coupled with kernel fusing leads to 15–30 percent performance gains for 64-point FFT and 256-point FFT
Results

- clFFT is an excellent implementation
- genFFT beats clFFT for:
  - 8, 16, and 32-FFT by more than 1.5x
  - 8k-FFT by more than 2x
  - But penalty for each additional kernel
    - 64 (8x8), 2k (32x8x8), and so on
- clFFT much better at fusing FFTs
  - Big penalty at 8k
- genFFT supports cl_half
  - Additional performance gains
  - There's still room for improvement

Relative performance of genFFT vs clFFT on Intel® HD Graphics 530 capped @ 750 MHz
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Conclusion

• Can be tuned for
  • Variable SIMD size
  • Available register space
  • Compiler ability to promote private memory to registers
  • Any combination of global-local sizes
• Avoids use of shared local memory and barriers
  • Reduced code complexity
  • Potentially better performance portability
• Cache-friendly implementation
• Penalty from enqueueing many kernels
Future Work

• cl_half performance improvements
  • Currently reading 1x32-bit quantities per work item while optimum is 4x32-bit

• Investigate other kernel fusing methods
  • Device-side enqueue

• Intel® Processor Graphics improvement opportunity
  • Optimize execution of pipelines of kernels that reuse buffers

• Expand work to any size 1D FFT and 2D FFT
References

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